

GEOMETRIC STRUCTURE OF MULTI-STATE OAM-CARRYING BEAMS

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A scalar, harmonic beam carrying orbital angular momentum (OAM) in several states can be represented as a multidimensional isotropic oscillator within a functional space defined by the orthonormal basis $e^{il\phi}$ where l and ϕ denote the OAM mode index and the azimuthal angle, respectively. The number of non-trivial OAM modes, say L , determines the dimension of this space and the optical field is then viewed as an LD vector. Within this framework, the optical field evaluated at a fixed radius and transverse plane, describes a trajectory that forms what is termed an *orbitalization ellipse* (OE) [1,2]. This nomenclature draws an analogy to the polarization ellipse used to describe the geometric structure of vector beams. The properties of the OE can be extracted from the polar Fourier series of the field. Figure 1 (A) shows the OE of superposition of three Laguerre-Gaussian (LG) modes in the Rayleigh plane at a fixed radius.

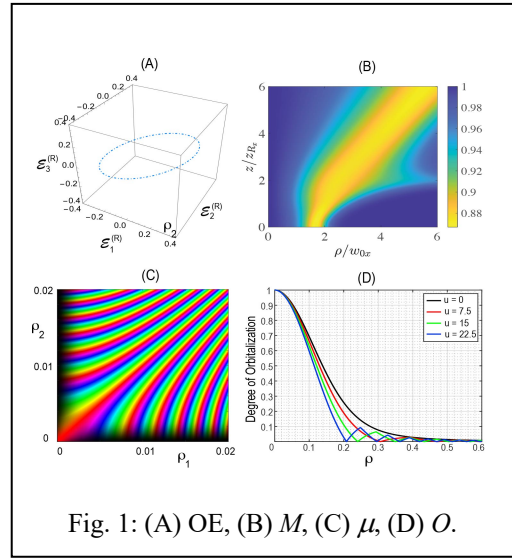


Fig. 1: (A) OE, (B) M , (C) μ , (D) O .

The analysis in [1,2] to a vectorial beam carrying OAM can be made via the $L \times 2$ polarization-orbitalization tensor (POT) [3] with rows (columns) accounting for L orbitalization (two polarization) states. On performing the Singular Value Decomposition of the POT 2D polarization and LD orbitalization projections can be accessed and the two corresponding ellipses (in 2D for all OAM states and in LD for both polarization states) can be deduced. The degree of mixing M of the two projections (at fixed radius and cross-section) can then be derived. Figure 1(B) shows the evolution of M of the three LG modes in vacuum. The analysis of the POT can also be extended to two radii [4]. Figure 1(C) shows the two-radii degree of similarity μ of the three LG modes in the polarization cross-section, being identical in form to the degree of coherence of random beams. This complex-valued quantity is shown with opacity for magnitude and color for phase.

The extension of theory in [1] to random beams was made in [5]. In the OAM space the beams' cross-spectral density can be split into those for pure and mixed states. While the OE can be extracted from the pure portion, the degree of orbitalization O can then be defined to quantify the ratio of the weight of the pure states to that in all states (similarly to the degree of polarization of 2D, 3D vector fields). Figure 1(D) shows O vs. radius in the TGSM source for several values of the twist parameter.

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