Optimal beam displacement measurements using high-order structured light modes

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Measurement of optical beam displacement is essential in various technological applications, such as atomic force microscopy [1, 2], optical tweezers [3], satellite alignment [4], and single-molecule tracking in biology [5]. The standard arrangement for measuring small angular displacements uses a lens followed by a split detector in the lens focal plane [1]. Although simple, this scheme reaches about 64% of the maximum theoretical precision [6] and is essentially applicable only to the fundamental Gaussian mode.

One way to overcome this limitation is to employ spatially multimode light, in which one of the modes is prepared in a squeezed state for detection [7, 8]. Another efficient approach, still within the Gaussian context, is to detect the first-order Hermite–Gaussian mode ($HG_{1,0}$) of the displaced beam via homodyne detection in that specific mode [9, 10]. In a more general scenario, when the probe beam is prepared in a high-order mode $HG_{m,0}$, the minimum measurable horizontal displacement is reduced by a factor $\sqrt{2m+1}$ [11]; in this case, homodyne detection requires a local oscillator in a superposition of $HG_{m-1,0}$ and $HG_{m+1,0}$.

We present an interferometric technique to measure lateral and angular displacements with optimal sensitivity in structured light beams, exploiting the inversion symmetry of high-order Hermite–Gaussian (HG) and Laguerre–Gaussian (LG) modes. Small displacements induce crosstalk into modes of opposite parity, which we read out optimally with an interferometric parity sorter. Our method achieves the same metrological gain improvement [11] but requires only intensity detection and does not rely on quantum light, high-order local oscillators, or homodyne detection. Compared to the fundamental Gaussian mode, we achieved SNR gains of up to 41 with HG modes (order $m = 20$) and 21 with LG modes (order $m = 10$). Our results confirm the optimality of LG modes for small-displacement sensing (consistent with quantum Fisher information) and show that the asymmetry of HG modes allows one to infer the displacement direction, paving the way for compact, high-precision metrology and alignment based on structured light.

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