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Efficiently checking physical realizability of experimental Mueller matrices

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Mueller polarimetry (MP) is an optical technique that is used for the characterization of polarimetric properties of a studied media. The idea is to measure changes in polarization states of incident light upon interaction with a sample. These changes are represented in the form of Mueller matrices (MM) which are real 4-by-4 matrices defining the corresponding transformation of the Stokes vectors.

In the context of biomedical imaging applications, a single "image" contains one MM per pixel (420k pixels in our case), and the objective is to process these images at the video rate. This poses a challenge: develop efficient algorithms for processing Mueller matrices allowing to perform calculations for hundreds of thousands of matrices in a fraction of a second on commodity hardware. In our recent paper [1] we addressed this challenge for the natural first step: checking physical realizability of the obtained matrices.

From purely mathematical standpoint, the problem is simple: one has to check if the corresponding coherence matrix (whose entries are linear combinations of the entries of MM) is positive semi-definite. However, one cannot just apply the standard tools to this problem because the classical efficient algorithms of numerical linear algebra are designed for large matrices, and 4-by-4 matrices are "below the radar", so the standard tools are not sufficiently efficient for them. On the other hand, the linear transformation used to obtain the coherence matrix from MM has rich structure coming from Pauli matrices, and it is natural to exploit the structure to speed up the calculations. Building upon these observations, we were able to achieve 4-fold speed up over the direct application of the state of the art numerical solvers.

[1]. T. Novikova, A. Ovchinnikov, G. Pogudin, J. Ramella-Roman, Bioinformatics, **40**(7), btae348 (2024)